# Mathematics

## Limit & continuity

**Properties of Limits :-**

These properties require that the limit of f(x) and g(x) exist.

$$\lim_{x\to a}[cf(x)] = c\lim_{x\to a} f(x)$$

$$\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$$

$$\lim_{x\to a}[f(x)g(x)] = \lim_{x\to a} f(x) \lim_{x\to a} g(x)$$

$$\lim_{x\to a}[f(x)/g(x)] = \lim_{x\to a} f(x)/\lim_{x\to a} g(x) \quad \text{if } \lim_{x\to a} g(x) \neq 0$$

$$\lim_{x\to a}[f(x)]^n = \lim_{x\to a}[f(x)]^n$$

**Limit Evaluation Method**: - Factor and cancle

$$\lim_{x\to -3} \frac{x^2 - x - 12}{X^2 + 3x} = \lim_{x\to -3}(x+3)(x-4)/x(x+3) = \lim_{x\to -3}(x-4)/x = 7/3$$

**L'hopital's Rule :-**

If $\lim_{x\to a} f(x)/g(x) = 0/0$ or $\pm\infty/\pm\infty$ then $\lim_{x\to a} f(x)/g(x) = \lim_{x\to a} f'(x)/g'(x)$

**Limit Evaluation At +, - , ∞ :-**

$\lim_{x\to\infty} e^x = \infty$ and $\lim_{x\to -\infty} e^x = 0$

$\lim_{x\to\infty} \ln(x) = \infty$ and $\lim_{x\to 0} \ln(x) = -\infty$

If r > 0 then $\lim_{x\to\infty} C/x' = 0$

If r>0 & $x^r$ is real for x<0 then $\lim_{x\to -\infty} C/x^r = 0$

$\lim_{x\to\pm\infty} x^r = \infty$ for even r

$\lim_{x\to\infty} x^r = \infty$ & $\lim_{x\to -\infty} x^r = -\infty$ for odd r

## VECTOR

**Modules of Vector :-**

$$\vec{a} = |\vec{a}| = \text{mod}\,\vec{a} = \text{mod}\,\vec{a} = a$$

module is also known as absolute value

Zero Vector :- $|\vec{a}| = a = 0$

Unit vector :- $\hat{a} = \vec{a}/|\vec{a}|$

**Properties of vector addition :-**

Commutative Law:- $\overline{a} + \overline{b} = \overline{b} + \overline{a}$

Associative Law :- $\overline{a}+\left(\overline{b}+\overline{c}\right)=\left(\overline{a}+\overline{b}\right)+\overline{c}$
Zero vector is an additive identity. $\overline{a}+\overline{0}=\overline{a}=\overline{0}+\overline{a}$
Additive Inverse:- $\overline{a}+\left(-\overline{a}\right)=\overline{0}=\left(-\overline{a}\right)+\overline{a}$

**Laws of scalar multiplication of a vector :-**

Associative Law:- If $\vec{a}$ is a vector and m, n are scalar then

$$m(n\ \overline{a})=n(mn)\overline{a}$$

Distributive Law :- If $\vec{a}$ is a vector and m, n are scalar then

$$(m+n)\overline{a}=m\overline{a}+n\overline{a}$$

**Section Formula :-**

When P intersect AB internally

$$\overline{r}=m\overline{b}+n\overline{a}/m+n$$

When P intersect AB externally

$$\overline{r}=m\overline{b}-n\overline{a}/m-n$$

**Mid-Point Formula :-**

$$\overline{r}=\frac{\overline{a}+\overline{b}}{2}$$

**Two Dimensional Unit Vector :-**

$$\overline{r}=\sqrt{x^2+y^2},\hat{r}=\overline{r}/|\overline{r}|=\overline{r}/r=xi+yj/\sqrt{x^2+y^2}$$

**Three Dimensional Unit Vector :-**

$$\overline{r}=x\hat{i}+y\hat{j}+z\hat{k}$$

$$r=\sqrt{x^2+y^2+z^2}$$

$$\hat{r}=x\hat{i}+y\hat{j}+z\hat{k}/\sqrt{x^2+y^2+z^2}$$

**Properties of Sclar :-**

Commutative law :- $\overline{a}.\overline{b}=\overline{b}.\overline{a}$
Associative Law does not hold.
Distributive Law:- $\overline{a}.\left(\overline{b}+\overline{c}\right)=\overline{a}.\overline{b}+\overline{a}.\overline{c}$

**Orthogonal Unit Vectors Triod :-**

$$\hat{i}.\hat{i}=1.1\cos 0^o=1=i^2$$

$$\hat{j}.\hat{j}=1.1\cos 0^o=1=j^2$$

$$\hat{k}.\hat{k}=1.1\cos 0^o=1=k^2$$

$$\hat{i}.\hat{j}=1.1\cos 90^o=0=\hat{j}.\hat{i}$$

$$\hat{j}.\hat{k}=1.1\cos 90^o=0=\hat{k}.\hat{j}$$

$$\hat{k}.\hat{i}=1.1\cos 90^o=0=\hat{i}.\hat{k}$$

**Distance Between Two Points:-**

$$PQ = \left|\vec{PQ}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

# DIFFERENTIAL EQUATION

**Logarithm Properties :-**

If $y = \log_b x$ then $b^y = x$

$\log_b b = 1$ and $\log_b 1 = 0$

$\log_b b^x = x$

$\log_a x = \log_b x / \log_b a$

$\log_b (x^r) = r \log_b x$

$\log_b (xy) = \log_b x + \log_b y$

$\log_b (x / y) = \log_b x - \log_b y$

**Properties of Complex Number :-**

$i = \sqrt{-1}$

$i^2 = -1$

$\sqrt{-a} = i\sqrt{a}, a >= 0$

$(a + bi) + (c + di) = a + c + (b + d)i$

$(a + bi) - (c + di) = a - c + (b - d)i$

$(a + bi)(c + di) = ac - bd + (ad + bc)i$

$(a + bi)(a - bi) = a^2 + b^2$

$|a + bi| = \sqrt{a^2 + b^2}$

$(a + bi) = a - bi$

$(a + bi)(a + bi) = |a + bi|^2$

$$\frac{1}{(a + bi)} = \frac{(a - bi)}{(a + bi)(a - bi)} = \frac{a - bi}{a^2 + b^2}$$

**Absolute Value :-**

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a \leq 0 \end{cases}$$

$|a| = |-a|$

$|a| \geq 0$

$|ab| = |a||b|$

$|a/b| = |a|/|b|$

$|a+b| \leq |a| + |b|$

# DIFFERENTIATION AND INTEGRATION

**Derivative Definitions :-**

$$d/dx\{f(x)\} = f'(x) = \lim_{h \to o} \frac{(f(x+h) - f(x))}{h}$$

**Basic Properties :-**

$\{cf(x)\}' = c\{f'(x)\}$

$\{f(x) \pm g(x)\}' = f'(x) \pm g'(x)$

$d/dx(c) = 0$

**Mean Value Theorem :-**

If f is differentiable on the interval (a, b) and continuous at the end points there exists a $C \, \varepsilon \, (\mathrm{a,b})$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

**Product Rule :-**

$\{f(x)g(x)\}' = f(x)' g(x) + f(x)g(x)'$

$$d \frac{(f(x))}{(g(x))} = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

**Power Rule :-**

$d/dx(x^n) = nx^{n-1}$

**Chain Rule :-**

$d/dx[f\{g(x)\}] = f[g(x)]g'(x)$

**Common Derivatives :-**

$d/dxf(x) = 1$

$d/dx(\sin x) = \cos x$

$d/dx(\cos x) = -\sin x$

$d/dx(\tan x) = \sec^2 x$

$d/dx(\sec x) = \sec x \tan x$

$d/dx(\cos ecx) = -\cos ec\, x \cot x$

$d/dx(\cot x) = -\cos ec^2 x$

$d/dx(\sin^{-1} x) = 1/\sqrt{1-x^2}$

$d/dx(\cos^{-1} x) = 1/\sqrt{1-x^2}$

$d/dx(\tan^{-1} x) = 1/\sqrt{1+x^2}$

$d / dx\left(a^{x}\right) = a^{x} \ln(a)$
$d / dx\left(e^{x}\right) = e^{x}$
$d / dx(\ln(x)) = 1 / x, \quad x>0$
$d / dx(\log x) = 1 / x$
$d / dx\{\log_{a}(x)\} = 1 / x \ln(a)$

**Chain Rule and Other Examples :-**

$d / dx\left([f(x)]^{n}\right) = n[f(x)]^{n-1} f'(x)$
$d / dx\left(e^{f(x)}\right) = f'(x) e^{f(x)}$
$d / dx(\ln[f(x)]) = f'(x) / f(x)$
$d / dx(\sin[f(x)]) = f'(x)\cos[f(x)]$
$d / dx(\cos[f(x)]) = -f'(x)\sin[f(x)]$
$d / dx(\tan[f(x)]) = -f'(x)\sec^{2}[f(x)]$
$d / dx(\sec[f(x)]) = f'(x)\sec[f(x)]\tan[f(x)]$
$d / dx\left(\tan^{-1}[f(x)]\right) = f'(x) / 1 + [f(x)]^{2}$
$d / dx\left(f(x)^{g(x)}\right) = f(x)^{g(x)}\left(g(x) f'(x) / f(x) + \ln f(x) g'(x)\right)$

**Definite Integral Definitions :-**

$$\int_{a}^{b} f(x) dx = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_{k}) \Delta x$$

Where $\Delta x = \frac{b-a}{n}$ and $x_{k} = a + k\Delta x$

**Fundamental Theoram of Calculus :-**

$$\int_{a}^{b} f(x) dx = [f(x)]_{a}^{b} = f(b) = f(a)$$

where f is continuous on [a, b] and f' = f

**Integration Properties :-**

$$\int_{a}^{b} cf(x) dx = c\int_{a}^{b} f(x) dx$$
$$\int_{a}^{b} f(x) \pm g(x) dx = \int_{a}^{b} f(x) dx \pm \int_{a}^{b} g(x) dx$$
$$\int_{a}^{b} f(x) = 0 \text{ and } \int_{a}^{b} f(x) dx = -\int_{b}^{a} f(x) dx$$
$$\int_{a}^{b} f(x) + \int_{b}^{c} f(x) = \int_{a}^{c} g(x) dx \qquad (a < b < c)$$

**Approximating Definite Integrals :-**

Left hand and right hand rectangle approximation

$$L_{n} = \Delta x \sum_{k=0}^{n-1} f(x_{k})$$
$$R_{n} = \Delta x \sum_{k=1}^{n-1} f(x_{k})$$

## Formula Booklet

**Mid Point Rule :-**

$$M_n = \Delta x \sum_{k=0}^{n-1} f\left(\frac{x_k + x_{k+1}}{2}\right)$$

**Integration by Substitutions :-**

$$\int_a^b f\{g(x)\} g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$$

where u = g(x) and du = g'(x)dx

**Common Integrals:-**

1. $\int 1dx = x + c$
2. $\int adx = ax + c$ Where a is any constant.
3. $\int x^n dx = \frac{x^{n+1}}{n+1} + c$
4. $\int [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1} + c$
5. $\int \frac{1}{x} dx = \ln x + c$
6. $\int \frac{f'(x)}{f(x)} dx = \ln f(x) + c$
7. $\int a^x dx = \frac{a^x}{\ln x} + c$
8. $\int a^{f(x)} dx = \frac{a^{f(x)}}{\ln a} + c$
9. $\int e^x dx = e^x + c$
10. $\int e^{f(x)} dx = e^{f(x)} + c$
11. $\int af(x)dx = a\int f(x)$
12. $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$
13. $\int f(x).g(x)dx = f(x)\left(\int g(x)dx\right) - \left[f'(x)\left(\int g(x)dx\right)\right]dx$
14. $\int \ln xdx = x(\ln x - 1) + c$
15. $\int \sin xdx = -\cos x + c$
16. $\int \cos xdx = \sin x + c$
17. $\int \tan xdx = \ln \sec x + c$ or $-\ln \cos x + c$
18. $\int \cot xdx = \ln \sin x + c$
19. $\int \sec xdx = \ln(\sec x + \tan x) + c$ or $\ln \tan\left(\frac{x}{2} + \frac{\pi}{4}\right) + c$
20. $\int \cos xdx = \ln(\cos x - \cot x) + c$ or $\ln \tan \frac{x}{2} + c$

21. $\int \sec^2 xdx = \tan x + c$
22. $\int \cos^2 xdx = -\cot x + c$
23. $\int \sec x \tan xdx = \sec x + c$
24. $\int \cos x \cot xdx = -\cos x + c$
25. $\int \sinh xdx = \cosh x + c$
26. $\int \cosh xdx = \sinh x + c$
27. $\int \tanh xdx = \ln \cosh x + c$
28. $\int \coth xdx = \ln \sinh x + c$
29. $\int \sec hxdx = \tan^{-1}(\sinh x) + c$
30. $\int \cosh xdx = -\coth h^{-1}(\cosh x)$
31. $\int \sec h^2 xdx = \tanh x + c$
32. $\int \cosh^2 xdx = -\coth x + c$
33. $\int \sec hx \tanh xdx = -\sec hx + c$
34. $\int \cosh x \coth xdx = -\cosh x + c$
35. $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} + c$ or $\cos^{-1} \frac{x}{a} + c$
36. $\int \frac{1}{\sqrt{x^2 - a^2}} dx = \cosh^{-1} \frac{x}{a} + c$ or $\ln\left(x + \sqrt{x^2 - a^2}\right) + c$
37. $\int \frac{1}{\sqrt{x^2 + a^2}} dx = \sinh^{-1} \frac{x}{a} + c$ or $\ln\left(x + \sqrt{x^2 + a^2}\right) + c$
38. $\int \frac{1}{a^2 - x^2} dx = \frac{1}{a} \tanh^{-1} \frac{x}{a} + c$ or $\frac{1}{2a} \ln\left(\frac{x - a}{x + a}\right) + c$
39. $\int \frac{1}{x^2 - a^2} dx = -\frac{1}{a} \coth^{-1} \frac{x}{a} + c$ or $\frac{1}{2a} \ln\left(\frac{x - a}{x + a}\right) + c$
40. $\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$
41. $\int \frac{1}{x\sqrt{a^2 - x^2}} dx = -\frac{1}{a} \sec h^{-1} \frac{x}{a} + c$ or $-\frac{1}{a} \ln\left(\frac{a + \sqrt{a^2 - x^2}}{x}\right) + c$
42. $\int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \sec^{-1} \frac{x}{a} + c$
43. $\int \frac{1}{x\sqrt{x^2 + a^2}} dx = -\frac{1}{a} \cosh^{-1} \frac{x}{a} + c$ or $-\frac{1}{a} \ln\left(\frac{a + \sqrt{x^2 + a^2}}{x}\right) + c$
44. $\int \sqrt{a^2 - x^2} dx = \frac{1}{2} x\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + c$

45. $\int \sqrt{x^2 - a^2}\,dx = \frac{1}{2}x\sqrt{x^2 - a^2} - \frac{a^2}{2}\cosh^{-1}\frac{x}{a} + c$ or
$\frac{1}{2}x\sqrt{x^2 - a^2} - \frac{a^2}{2}\ln\left(x + \sqrt{x^2 - a^2}\right) + c$

46. $\int \sqrt{x^2 + a^2}\,dx = \frac{1}{2}x\sqrt{x^2 + a^2} - \frac{a^2}{2}\sinh^{-1}\frac{x}{a} + c$ or
$\frac{1}{2}x\sqrt{x^2 + a^2} + \frac{a^2}{2}\ln\left(x + \sqrt{x^2 + a^2}\right) + c$

47. $\int e^{ax}\sin(bx + c)dx = \frac{e^{\alpha x}}{a^2 + b^2}\left[a\sin(bx + c) + b\cos(bx + c)\right]$

48. $\int e^{ax}\cos(bx + c)dx = \frac{e^{\alpha x}}{a^2 + b^2}\left[a\cos(bx + c) + b\sin(bx + c)\right]$

49. $\int \sin mx\cos nx dx = -\frac{\cos(m+n)x}{2(m+n)} - \frac{\cos(m-n)x}{2(m-n)} + c$

50. $\int \sin mx\sin nx dx = -\frac{\sin(m+n)x}{2(m+n)} + \frac{\sin(m-n)x}{2(m-n)} + c$

51. $\int \cos mx\cos nx dx = \frac{\sin(m+n)x}{2(m+n)} + \frac{\sin(m-n)x}{2(m-n)} + c$

52. $\int \sin^{-1} x dx = x\sin^{-1} x + \sqrt{1 - x^2} + c$

53. $\int \cos^{-1} x dx = x\cos^{-1} x - \sqrt{1 - x^2} + c$

54. $\int \tan^{-1} x dx = x\tan^{-1} x - \frac{1}{2}\ln\left(1 + x^2\right) + c$

55. $\int \cot^{-1} x dx = x\cot^{-1} x + \frac{1}{2}\ln\left(1 + x^2\right) + c$

56. $\int \sec^{-1} x dx = x\sec^{-1} x - \ln\left(x + \sqrt{x^2 - 1}\right) + c$

57. $\int \cos^{-1} x dx = x\cos^{-1} x - \ln\left(x + \sqrt{x^2 - 1}\right) + c$

58. $\int \frac{1}{a + b\sin x}dx = \frac{1}{\sqrt{a^2 - b^2}}\ln\left(\frac{a\tan\frac{x}{a} + b - \sqrt{b^2 - a^2}}{a\tan\frac{x}{a} + b + \sqrt{b^2 - a^2}}\right) + c$ if $a^2 < b^2$

59. $\int \frac{1}{a + b\cos x}dx = \frac{2}{\sqrt{a^2 - b^2}}\tan^{-1}\left(\sqrt{\frac{a - b}{a + b}}\tan\frac{x}{2}\right) + c$ if $a^2 > b^2$

60. $\int \frac{1}{a + b\cos x}dx = \frac{2}{\sqrt{a^2 - b^2}}\ln\left(\frac{\sqrt{b + a} + \tan\frac{x}{2}\sqrt{b - a}}{\sqrt{b + a} - \tan\frac{x}{2}\sqrt{b - a}}\right) + c$ if $a^2 < b^2$

61. $\int \frac{1}{a + b\sin x}dx = \frac{1}{\sqrt{a^2 + b^2}}\ln\left(\frac{\sqrt{a^2 + b^2} + a\tanh\frac{x}{2} - b}{\sqrt{a^2 + b^2} - a\tanh\frac{x}{2} + b}\right) + c$

62. $\int \frac{1}{a+b\cosh x}\,dx = \frac{\sqrt{a+b}+\sqrt{a-b}\tanh\frac{x}{2}}{\sqrt{a+b}-\sqrt{a-b}\tanh\frac{x}{2}}+c$ if a > b

63. $\int \frac{1}{a+b\cosh x}\,dx = \frac{2}{\sqrt{b^2-a^2}}\tan^{-1}\sqrt{\frac{b-a}{b+a}}\tanh^{-1}\frac{x}{2}+c$ if a < b

# PROBABILITY

$$P = \frac{\text{Favaurable Condition}}{\text{Total Condition}}$$

Total probability is always 1.

**Theorem of total probability :-**

$p(A \cup B) = p(A) + p(B) - p(A \;\; B)$ or

$p(A \text{ or } B) = p(A) + p(B) - p(A \,\&\, B)$

**Theorom of total probability :-**

$p(A \cup B) = p(A) + p(B) - p(A \cap B)$ or

$p(A \text{ or } B) = p(A) + p(B) - p(A \,\&\, B)$

**If condition A & B are Independent :-**

$p(A \cap B) = p(A).p(B)$

**Conditional Probability :-**

$$p(A/B) = \frac{n(A \cap B)/n(s)}{n(B)/n(s)}$$

$p(A \cap B)/\,p(B)$

or p(A/B) = p(A/B).p(B)

## <u>General Formulas of Trigonometry</u>

**Trigonometric Substitution :-**

| Expression | Substitution |
|---|---|
| $\sqrt{A^2-x^2}$ | x = Asinθ<br>dx = Acosθ $d$θ |
| $\sqrt{x^2-a^2}$ | x = a secθ<br>dx = asecθ tanθ $d$θ |
| $\sqrt{a^2+x^2}$ | x = atanθ<br>dx = asec$^2$θ $d$θ |

## Formula Booklet

**Trig Function Range :-**

$$-1 \le \sin\theta \le 1$$
$$-1 \le \cos\theta \le 1$$
$$-\infty < 1 < \infty$$
$$\cos ec\theta \ge 1 \text{ and } \cos ec\theta \le -1$$
$$\sec\theta \le 1 \text{ and } \sec\theta \le -1$$
$$-\infty \le \cot\theta \le \infty$$

**Inverse Trig Function Range :-**

$$-\pi/2 \le \sin^{-1} x \le x/2$$
$$0 \le \cos^{-1} x \le \pi$$
$$-\pi/2 \le \tan^{-1} \le \pi/2$$

**Periodic Identities :-**

$$\sin(\theta + 2\pi n) = \sin\theta$$
$$\cos(\theta + 2\pi n) = \cos\theta$$
$$\tan(\theta + \pi n) = \tan\theta$$
$$\cos ec(\theta + 2\pi n) = \cos ec\theta$$
$$\sec(\theta + 2\pi n) = \sec\theta$$
$$\cot(\theta + \pi n) = \cot\theta$$

**Double Angle Identities :-**

$$\sin(2u) = 2\sin u \cos u$$
$$\cos(2u) = \cos^2 u - \sin^2 u$$
$$= 2\cos^2 u - 1$$
$$= 1 - 2\sin^2 u$$
$$\tan(2u) = \frac{2\tan u}{1 - \tan^2 u}$$

**Half Angle Identities :-**

$$\sin\left(\frac{a}{2}\right) = \pm\sqrt{\frac{(1-\cos a)}{2}}$$
$$\cos\left(\frac{a}{2}\right) = \pm\sqrt{\frac{(1+\cos a)}{2}}$$
$$\tan\left(\frac{a}{2}\right) = \frac{1-\cos a}{\sin a} = \frac{\sin a}{1+\cos a}$$

**Product-to-Sum Identities :-**

$$\sin u \sin v = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$$
$$\cos u \cos v = \frac{1}{2}[\cos(u-v) + \sin(u+v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$$
$$\cos u \sin v = \frac{1}{2}[\sin(u+v) - \sin(u-v)]$$

**Sum-to-Product Identities :-**

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$
$$\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$
$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$
$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

**Even/Odd Identities :-**

$$\sin(-x) = -\sin x \cos(-x) = \cos x \tan(-x) = -\tan x$$
$$\cos(-x) = -\cos x \sec(-x) = \sec x \cot(-x) = -\cot x$$

**Sum-Difference Identities :-**

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$
$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$
$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

**Law of Cosines :-**

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$
$$b^2 = a^2 + c^2 - 2ac\cos\beta$$
$$c^2 = a^2 + b^2 - 2bc\cos\gamma$$

**Law of Sines :-**

$$\sin\frac{\alpha}{a} = \sin\frac{\beta}{a} = \sin\frac{\gamma}{a}$$

**Law of Tangent :-**

$$\frac{a-b}{a+b} = \frac{\tan[1/2(\alpha-\beta)]}{\tan[1/2(\alpha+\beta)]}$$
$$\frac{b-c}{b+c} = \frac{\tan[1/2(\beta-\gamma)]}{\tan[1/2(\beta+\gamma)]}$$
$$\frac{a-c}{a+c} = \frac{\tan[1/2(\alpha-\gamma)]}{\tan[1/2(\alpha+\gamma)]}$$